1.3 Resistors and Ohm’s Law

Overview

Resistance is a property of all materials – this property characterizes the loss of energy associated with passing an electrical current through some conductive element. Resistors are circuit elements whose characteristics are dominated by this energy loss. Since energy is always lost when current is passed through an electrical circuit element, all electrical elements exhibit resistive properties which are characteristic of resistors. Resistors are probably the simplest and most commonly used circuit elements.

Before beginning this chapter, you should be able to:

- Perform basic algebra
- State the passive sign convention from memory (Chapter 1.1)
- Determine the power absorbed or generated by an circuit element, based on the current and voltage provided to it (Chapter 1.2)

After completing this chapter, you should be able to:

- State Ohm’s Law from memory
- Use Ohm’s Law to perform voltage and current calculations for resistive circuit elements

This chapter requires:

- N/A

All materials impede the flow of current through them to some extent. Essentially, this corresponds to a statement that energy is always lost when transferring charge from one point in a circuit to another – this energy loss is generally due to heat generation and dissipation. The amount of energy required to transfer current in a particular element is characterized by the resistance of the element. When modeling a circuit, this resistance is represented by resistors. The circuit symbol for a resistor is shown in Figure 1. The value of resistance is labeled in Figure 1 as \( R \). \( i \) in Figure 1 is the current flowing through the resistor and \( v \) is the voltage drop across the resistor, caused by the energy dissipation induced by the resistor. The units of resistance are ohms, abbreviated \( \Omega \).

The relationship between voltage and current for a resistor is given by Ohm’s Law:

\[
v(t) = Ri(t) \tag{1}
\]

where voltage and current are explicitly denoted as functions of time. Note that in Figure 1, the current is flowing from a higher voltage potential to a lower potential, as indicated by the polarity (+ and -) of the voltage and the arrow indicating direction of current flow. The relative polarity between voltage and current for a resistor must be as shown in Figure 1; the current enters the node at which
the voltage potential is highest. Values of resistance, $R$, are always positive, and resistors always absorb power.

Note: The voltage-current relationship for resistors always agrees with the passive sign convention. Resistors always absorb power.

Figure 1. Circuit symbol for resistor.

Figure 2 shows a graph of $v$ vs. $i$ according to equation (1); the resulting plot is a straight line with slope $R$. Equation (1) thus describes the voltage-current relationship for a linear resistor. Linear resistors do not exist in reality – all resistors are nonlinear, to some extent. That is, the voltage-current relationship is not exactly a straight line for all values of current (for example, all electrical devices will fail if enough current is passed through them). Figure 3 shows a typical nonlinear voltage-current relationship. However, many nonlinear resistors exhibit an approximately linear voltage-current characteristic over some range of voltages and currents; this is also illustrated in Figure 3. We will assume for now that any resistor we use is operating within a range of voltages and currents over which its voltage-current characteristic is linear and can be approximated by equation (1).

Note: We will consider only linear resistors. These resistors obey the linear voltage-current relationship shown in equation (1). All real resistors are nonlinear to some extent, but can often be assumed to operate as linear resistors over some range of voltages and currents.

Figure 2. Linear resistor voltage vs. current characteristic.
1.3 Resistors and Ohm’s Law

**Conductance:**

Conductance is an important quantity in circuit design and analysis. Conductance is simply the reciprocal of resistance, defined as:

\[
G = \frac{1}{R}
\]

(2)

The unit for conductance is siemens, abbreviated S. Ohm’s law, written in terms of conductance, is:

\[
i(t) = Gv(t)
\]

(3)

Some circuit analyses can be performed more easily and interpreted more readily if the elements’ resistance is characterized in terms of conductance.

**Reminder:**

In chapter 1.2, we characterized a current-controlled voltage source in terms of a parameter with units of ohms, since it had units of volts/amp. We characterized a voltage-controlled current source in terms of a parameter with units of siemens, since it had units of amps/volt.

**Power Dissipation**

Instantaneous power was defined by equation (3) in Chapter 1.1 as:

\[
P(t) = v(t) \cdot i(t)
\]

For the special case of a resistor, we can re-write this (by substituting equation (1) into the above) as:
1.3 Resistors and Ohm’s Law

\[ P(t) = Ri^2(t) = \frac{v^2(t)}{R} \]  

(4)

Likewise, we can write the power dissipation in terms of the conductance of a resistor as:

\[ P(t) = \frac{i^2(t)}{G} = Gv^2(t) \]  

(5)

Note: We can write the power dissipation from a resistor in terms of the resistance or conductance of the resistor and either the current through the resistor or the voltage drop across the resistor.

Practical resistors.

All materials have some resistance, so all electrical components have non-zero resistance. However, circuit design often relies on implementing a specific, desired resistance at certain locations in a circuit; resistors are often placed in the circuit at these points to provide the necessary resistance. Resistors can be purchased in certain standard values. Resistors are manufactured in a variety of ways, though most commonly available commercial resistors are carbon composition or wire-wound. Resistors can have either a fixed or variable resistance.

Fixed resistors provide a single specified resistance value and have two terminals, as shown in Figure 1 above. Variable resistors or potentiometers (commonly called “pots”) have three terminals, two are “fixed” and one is “movable”. The symbol for a variable resistor is shown in Figure 4. The resistance between two of the terminals – R_{23} in Figure 4 – of a variable resistor can be set as some fraction of the overall resistance of the device – R_{13} in Figure 4. The ratio of R_{23} to R_{13} is generally set by a dial or set screw on the side of the device.

![Figure 4. Schematic for variable resistor.](image)

Resistors which are physically large enough will generally have their resistance value printed directly on them. Smaller resistors generally will use a color code to identify their resistance value. The color coding scheme is provided in Figure 5. The resistance values indicated on the resistor will provide a
nominal resistance value for the component; the actual resistance value for the component will vary from this by some amount. The expected tolerance between the allowable actual resistance values and the nominal resistance is also provided on the resistor, either printed directly on the resistor or provided as an additional color band. The color coding scheme for resistor tolerances is also provided in Figure 5.

![Resistor Tolerance Code](image)

<table>
<thead>
<tr>
<th>Fourth Band</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>±20%</td>
</tr>
<tr>
<td>Silver</td>
<td>±10%</td>
</tr>
<tr>
<td>Gold</td>
<td>±5%</td>
</tr>
</tbody>
</table>

Resistance = \((10^A + B) \times 10^{Exponent \pm Tolerance}\)

Figure 5. Resistor color code.

Example:

A resistor has the following color bands below. Determine the resistance value and tolerance.

First band (A): Red
Second band (B): Black
Exponent: Orange
Tolerance: Gold

\[
\text{Resistance} = (20 + 0) \times 10^3 \pm 5\% = 20 \text{k}\Omega \pm 1\text{k}\Omega
\]