1.6.2 Mesh Analysis

Overview

In mesh analysis, we will define a set of mesh currents and use Ohm’s law to write Kirchoff’s voltage law in terms of these voltages. The resulting set of equations can be solved to determine the mesh currents; any other circuit parameters (e.g. voltages) can be determined from these currents.

Mesh analysis is appropriate for planar circuits. Planar circuits can be drawn in a single plane such that no elements overlap one another. Such circuits, when drawn in a single plane will be divided into a number of distinct areas; the boundary of each area is a mesh of the circuit. A mesh current is the current flowing around a mesh of the circuit.

Before beginning this chapter, you should be able to:

- Write Ohm’s law for resistive circuit elements (Chapter 1.3)
- Apply Kirchoff’s voltage and current laws to electrical circuits (Chapter 1.4)

After completing this chapter, you should be able to:

- Use mesh analysis techniques to analyze electrical circuits

This chapter requires:

- N/A

The steps used to in mesh analysis are provided below. The steps are illustrated in terms of the circuit of Figure 1.

![Figure 1. Example circuit.](image)
Step 1: Define mesh currents

In order to identify our mesh loops, we will turn off all sources. To do this, we
- Short-circuit all voltage sources
- Open-circuit all current sources.

Once the sources have been turned off, the circuit can be divided into a number of non-overlapping areas, each of which is completely enclosed by circuit elements. The circuit elements bounding each of these areas form the meshes of our circuit. The mesh currents flow around these meshes. Our example circuit has two meshes after removal of the sources, the resulting mesh currents are as shown in Figure 2.

Note:

We will always choose our mesh currents as flowing clockwise around the meshes. This assumption is not fundamental to the application of mesh analysis, but it will result in a special form for the resulting equations which will later allow us to do some checking of our results.

Figure 2. Example circuit meshes.

Step 2: Replace sources and identify constrained loops

The presence of current sources in our circuit will result in the removal of some meshes during step 1. We must now account for these meshes in our analysis by returning the sources to the circuit and identifying constrained loops.

We have two rules for constrained loops:
1. Each current source must have one and only one constrained loop passing through it.
2. The direction and magnitude of the constrained loop current must agree with the direction and magnitude of the source current.

For our example circuit, we choose our constrained loop as shown below. It should be noted that constrained loops can, if desired, cross our mesh loops – we have, however, chosen the constrained loop so that it does not overlap any of our mesh loops.
Step 3: Write KVL around the mesh loops

We will apply Kirchhoff’s voltage law around each mesh loop in order to determine the equations to be solved. Ohm’s law will be used to write KVL in terms of the mesh currents and constrained loop currents as identified in steps 1 and 2 above.

Note that more than one mesh current may pass through a circuit element. When determining voltage drops across individual elements, the contributions from all mesh currents passing through that element must be included in the voltage drop.

When we write KVL for a given mesh loop, we will base our sign convention for the voltage drops on the direction of the mesh current for that loop.

For example, when we write KVL for the mesh current \( i_1 \) in our example, we choose voltage polarities for resistors \( R_1 \) and \( R_4 \) as shown in the figure below – these polarities agree with the passive sign convention for voltages relative to the direction of the mesh current \( i_1 \).

From the above figure, the voltage drops across the resistor \( R_1 \) can then be determined as

\[
V_1 = R_1 i_1
\]

since only mesh current \( i_1 \) passes through the resistor \( R_1 \). Likewise, the voltage drop for the resistor \( R_4 \) is

\[
V_4 = R_4 (i_1 - i_2)
\]

since mesh currents \( i_1 \) and \( i_4 \) both pass through \( R_4 \) and the current \( i_2 \) is in the opposite direction to our assumed polarity for the voltage \( V_4 \).
Using the above expressions for $V_1$ and $V_4$, we can write KVL for the first mesh loop as:

$$-V_S + R_1 i_1 + R_4 (i_1 - i_2) = 0$$

When we write KVL for the mesh current $i_2$ in our example, we choose voltage polarities for resistors $R_4$, $R_2$, and $R_5$ as shown in the figure below – these polarities agree with the passive sign convention for voltages relative to the direction of the mesh current $i_2$. Please note that these sign conventions do not need to agree with the sign conventions used in the equations for other mesh currents.

Using the above sign conventions, KVL for the second mesh loop becomes:

$$R_4 (i_2 - i_1) + R_2 i_2 + R_5 (i_2 + I_S) = 0$$

Please note that the currents $i_2$ and $I_S$ are in the same direction in the resistor $R_5$, resulting in a summation of these currents in the term corresponding to the voltage drop across the resistor $R_5$.

**Notes:**

1. Assumed sign conventions on voltages drops for a particular mesh loop are based on the assumed direction of that loop’s mesh current.
2. The current passing through an element is the algebraic sum of all mesh and constraint currents passing through that element. This algebraic sum of currents is used to determine the voltage drop of the element.

**Step 4:** Solve the system of equations to determine the mesh currents of the circuit.

Step 5 will always result in $N$ equations in $N$ unknowns, where $N$ is the number of mesh currents identified in step 1. These equations can be solved for the mesh currents. Any other desired circuit parameters can be determined from the mesh currents.

The example below illustrates the above approach.
Example:

In the circuit below, determine the voltage drop, \( V \), across the 3\( \Omega \) resistor.

![Circuit diagram]

Removing the sources results in a single mesh loop with mesh current \( i_1 \), as shown below.

![Loop diagram]

Replacing the sources and defining one constrained loop per source results in the loop definitions shown below. (Note that each constrained loop goes through only one source and that the amplitude and direction of the constrained currents agrees with source.)

![Constrained loop diagram]

Applying KVL around the loop \( i_1 \) and using Ohm's law to write voltage drops in terms of currents:

\[
-3V + 7\Omega(i_1 + 1A) + 3\Omega(i_1 + 1A + 3A) + 4\Omega(i_1 + 3A) = 0 \implies i_1 = -2A
\]

Thus, the current \( i_1 \) is 2A, in the opposite direction to that shown. The voltage across the 3\( \Omega \) resistor is \( V = 3\Omega(i_1 + 3A + 1A) = 3\Omega(-2A + 3A + 1A) = 3(2A) = 6V \).
Alternate Approach to Constraint Loops: Constraint Equations

In the above examples, the presence of current sources resulted in a reduced number of meshes. Constraint loops were then used to account for current sources. An alternate approach, in which we retain additional mesh currents and then apply constraint equations to account for the current sources, is provided here. We use the circuit of the previous example to illustrate this approach.

Example: Determine the voltage, $V$, in the circuit below.

![Circuit Diagram]

Define three mesh currents for each of the three meshes in the above circuit and define unknown voltages $V_1$ and $V_3$ across the two current sources as shown below:

![Circuit Diagram]

Applying KVL around the three mesh loops results in three equations with five unknowns:

$$V_1 + 3\Omega \cdot (i_1 - i_3) + 7\Omega \cdot (i_1 - i_2) = 0$$

$$-3V + 7\Omega \cdot (i_2 - i_1) + V_3 = 0$$

$$-V_3 + 3\Omega \cdot (i_3 - i_1) + 4\Omega \cdot i_3 = 0$$

Two additional constraint equations are necessary. These can be determined by the requirement that the algebraic sum of the mesh currents passing through a current source must equal the current provided by the source. Thus, we obtain:

$$-i_2 + i_1 = 3A$$

$$-i_1 = 1A$$

Solving the five simultaneous equations above results in the same answer determined previously.
Clarification: Constraint loops

Previously, it was claimed that the choice of constraint loops is somewhat arbitrary. The requirements are that each source has only one constraint loop passing through it, and that the magnitude and direction of the constrained loop current be consistent with the source. Since constraint loops can overlap other mesh loops without invalidating the mesh analysis approach, the choice of constraint loops is not unique. The examples below illustrate the effect of different choices of constraint loops on the analysis of a particular circuit.

Example – version 1: Using mesh analysis, determine the current \( i \) through the 4\( \Omega \) resistor.

**Step 1: Define mesh loops**

Replacing the two current sources with open circuits and the two voltage sources with short circuits results in a single mesh current, \( i_1 \), as shown below.
Step 2: Constrained loops – version 1

Initially, we choose the constrained loops shown below. Note that each loop passes through only one source and has the magnitude and direction imposed by the source.

![Diagram of constrained loops]

Step 3: Write KVL around the mesh loops

Our example has only one mesh current, so only one KVL equation is required. This equation is:

$$-8V + 2\Omega(i_1 + 1A - 2A) + 4\Omega(i_1 - 2A) + 10V + 6\Omega(i_1) = 0$$

Step 4: Solve the system of equations to determine the mesh currents of the circuit.

Solving the above equation results in $i_1 = 0.667A$. The current through the $4\Omega$ resistor is then, accounting for the $2A$ constrained loop passing through the resistor, $i = i_1 - 2A = -1.333A$. 
Example – version 2:

In this version, we choose an alternate set of constraint loops. The alternate set of loops is shown below; all constraint loops still pass through only one current source, and retain the magnitude and direction of the source current.

Now, writing KVL for the single mesh results in:

\[-8V + 2\Omega(i_1 + 1A) + 4\Omega \cdot i_1 + 10V + 6\Omega(i_1 + 2A) = 0\]

Solving for the mesh current results in \(i_1 = -1.333A\); note that this result is different than previously. However, we determine the current through the 4Ω resistor as \(i = i_1 = -1.333A\), which is the same result as previously.

Note:

Choice of alternate constrained loops may change the values obtained for the mesh currents. The currents through the circuit elements, however, do not vary with choice of constrained loops.
Example – version 3:

In this version, we choose yet another set of constraint loops. These loops are shown below. Again, each loop passes through one current source and retains that source’s current direction and amplitude.

KVL around the mesh loop results in

\[-8V + 2\Omega \cdot i_i + 4\Omega(i_i - 1A) + 10V + 6\Omega(i_i - 1A + 2A) = 0\]

Which results in \(i_i = -0.333A\). Again, this is different from the result from our first two approaches. However, the current through the 4Ω resistor is \(i = i_i - 1A = -1.333A\), which is the same result as previously.

Dependent Sources:

As with nodal analysis, the presence of dependent sources does not significantly alter the overall mesh analysis approach. The primary difference is simply the addition of the additional equations necessary to describe the dependent sources. We discuss the analysis of circuits with dependent sources in the context of the following examples.
Example: Determine the voltage \( V \) in the circuit below.

![Circuit Diagram](image)

Shorting both of the voltage sources in the circuit above results in two mesh currents. These are shown in the figure below.

![Mesh Currents Diagram](image)

Writing KVL around the two mesh loops results in

\[
-2V + 2\Omega \cdot i_1 + 3\Omega (i_1 - i_2) = 0 \\
2I_x + 3\Omega (i_2 - i_1) + 4\Omega \cdot i_2 = 0
\]

We have two equations and three unknowns. We need an additional equation to solve the system of equations. The third equation is obtained by writing the dependent source’s controlling current in terms of the mesh currents:

\[
I_x = i_1
\]

The above three equations can be solved to obtain \( i_1 = 0.4375\Omega \) and \( i_2 = 0.0625\Omega \). The desired voltage \( V = 4i_2 = 0.25V \).
Example: Write the mesh equations for the circuit shown below.

Mesh loops and constraint loops are identified as shown below:

Writing KVL for the two mesh loops results in:

\[ 4\Omega \cdot i_1 + 2\Omega(i_1 - 3V_x) + 12V = 0 \]
\[ -12V + 3\Omega(i_2 - 3V_x) + 5\Omega \cdot i_2 = 0 \]

Writing the controlling voltage \( V_x \) in terms of the mesh currents results in:

\[ V_x = 5\Omega \cdot i_2 \]

The above consist of three equations in three unknowns, which can be solved to determine the mesh currents. Any other desired circuit parameters can be determined from the mesh currents.