Overview

In this lab, we will experimentally explore the characteristics of resistors. Resistance is measured using an ohmmeter; this measured resistance is compared to a voltage-current characteristic curve measured for the resistor. Some basic statistical data analysis methods are introduced briefly in this lab assignment. MATLAB commands used to perform basic statistical analysis of data are provided.

Nonlinear resistance is explored by requiring excessive power dissipation from a resistor, causing failure of the resistor.

In Part IV of this lab assignment, we create our first circuit which appears to “do” something which is readily perceivable without instrumentation. The circuit operates as a “dusk-to-dawn” light; the circuit turns a light on when the ambient light level goes below a certain level.

Before beginning this chapter, you should be able to:

- State Ohm’s law from memory (Chapter 1.3)
- Use Ohm’s law to perform voltage and current calculations for resistive circuit elements (Chapter 1.3)
- Use a digital multimeter to measure resistance and voltage (Lab 0)
- State KVL and KCL from memory (Chapter 1.4)
- Use color codes on resistors to determine the resistor’s nominal resistance (Chapter 1.3)

This lab exercise requires:

- Digilent Analog Parts Kit
- Breadboard
- Digital multimeters (DMMs), DC power supplies

After completing this chapter, you should be able to:

- Calculate the mean, median, and standard deviation values of a set of data
- Determine the least-squares best fit straight line approximating a set of data
- Calculate the correlation coefficient between a set of data and a line approximating the data
- Estimate resistance from measured voltage-current data
I. **Resistance Variations**

*General Discussion:*

Fixed resistors are fabricated with a nominal resistance value. Individual resistors will, in general, not have exactly this resistance – their actual resistance will vary from this nominal value to some extent. In this portion of the lab assignment, we will measure the resistance of several resistors which all have the same **nominal** resistance, in order to determine the actual resistor-to-resistor resistance variations. These variations will be compared with the manufacturer’s specified tolerances.

**Important Note:**

Since the resistance of individual resistors will vary from the value specified by the manufacturer, it is generally important, when incorporating a resistor in a circuit, to measure the resistor’s **actual** resistance and record that value in your laboratory notebook when you are constructing a circuit. It is possible that the variations in resistance can affect your experimental results.

We will use **statistical analyses** to in this section to examine variations in resistors’ resistances. Statistics are a way to quantify variations in results which are due to random effects such as variations in manufacturing, uncertainty in measured quantities, and measurement noise. In this section, we will briefly present some common statistical values used to characterize data which have some random component. We will also present MATLAB commands which can be used to determine these values.

**Mean value:**

If we have N sample values, \{y_1, y_2, \ldots, y_N\}, we can determine the mean of the values by the following expression:

\[
\bar{y} = \frac{1}{N} \left( y_1 + y_2 + y_3 + \cdots + y_N \right) = \frac{1}{N} \sum_{i=1}^{N} y_i
\]  

(1)

Thus, the mean can be determined by summing all the sample values and dividing by the total number values. It is essentially what we do when we determine an **average** of a series of values.
Median value:

The median value of a set of N sample values \( \{y_1, y_2, \ldots, y_N\} \) is the value which has the same number of samples above the value as there are below the value. This definition, however, does not necessarily uniquely determine the median value of a set of numbers. We will, therefore, refine our definition as follows:

- If N is an odd number, sort the data so that they are ordered from smallest to largest. The median value is then the middle data value.
- If N is an even number, again sort the data so that they are ordered from smallest to largest. There is no value exactly at the middle of the resulting sequence, so we choose the median value as the mean of the middle two numbers.

Standard deviation:

The standard deviation of a set of values \( \{y_1, y_2, \ldots, y_N\} \) can be thought of as the amount of “spread” that the numbers have from their mean value. Thus, the standard deviation provides an indication as to how closely grouped the values are around their mean. The definition of standard deviation that we will use is:

\[
s = \frac{1}{N-1} \sqrt{\sum_{i=1}^{N} (y_i - \bar{y})^2}
\]

where \( s \) is the standard deviation and \( \bar{y} \) is the mean of the values, as defined above. The positive square root is used in the calculation, so that \( s \) is always represented as a positive number. Some textbooks use an alternate version of the standard deviation; we will exclusively use equation (2). When using a software package to perform statistical analyses, always check the definition of standard deviation that is used.

MATLAB syntax:

If the variable \( y \) has been defined as a vector of numbers in the workspace, the following MATLAB syntax will calculate the mean, median, and standard deviation of the numbers.

Mean: \( \text{mean}(y) \)

Median: \( \text{median}(y) \)

Standard Deviation: \( \text{std}(y) \)

Pre-lab: None
Lab Procedures:

1. Pick five (5) 2.2KΩ resistors from your analog parts kit. Using an ohmmeter, measure the resistance of each of the resistors and record the values in your lab notebook. Calculate the mean, median, and standard deviation of the resistance values and record these quantities in your lab notebook. Convert these values to percentages of the nominal resistance, and record these results in your lab notebook. Determine the manufacturer’s tolerance on the resistance values of the resistance and comment on whether your resistors lie within the specified tolerance.

2. Repeat procedure 1 above with five (5) 47KΩ resistors.

II. Resistance from measured current-voltage data

General Discussion:

We have previously noted that the resistance of a component is the slope of the current vs. voltage curve for the component. In this part of the lab assignment, we will measure a current-voltage characteristic curve for a resistor and estimate a resistance from this data. We will compare this resistance from the resistance measured by an ohmmeter.

In order to experimentally determine the current-voltage characteristic for our resistor, we will use the circuit shown schematically in Figure 1. A variable voltage source will be used to apply the voltage across the resistor, \( v_s \). We will use DMMs to measure the resistor voltage drop and the current through the resistor, \( v_R \) and \( i_R \), respectively. By varying \( v_s \), we can measure a set of values for \( v_R \) and \( i_R \) and plot \( v_R \) vs \( i_R \), as shown in Figure 2(a). The slope of the line that “best fits” the measured data can then be used to estimate the component’s resistance, as shown in Figure 2(b).

Note:

Your power supply may display the voltage and/or current provided to the circuit. Do not use the values displayed by the power supply as the resistor voltage and current, \( v_R \) and \( i_R \). The values displayed by the power supply may differ from the resistor’s voltage and current due to non-ideal power supply effects, such as the power supply internal resistance.

![Figure 1. Part II circuit schematic.](image)
Least-squares curve fitting

Experimental data will always contain some uncertainty, so measured current-voltage data for a resistor will never lie exactly on a straight line – see, for example, Figure 2. Thus, the notion of a “best” straight-line approximation to the data is rather nebulous. It is simplest to draw by eye a straight line through the plotted data. This approach, while used fairly often, has the drawback that no two engineers are likely to draw the same straight line. Thus, we look for a more objective and readily quantifiable approach toward fitting a line to a set of measured data. One common approach toward determining a line which provides a “best fit” to the available data is least squares curve fitting. The basic idea behind the least-squares approach toward fitting a curve to data is as follows:

- We have a set of x, y data, where the x data points are \{x_1, x_2, \ldots, x_N\} and the y data points are \{y_1, y_2, \ldots, y_N\}

- Assume that a straight line will approximate the data. The equation for the straight line is

\[ y = mx + b \]  \hspace{1cm} (3)

where \( m \) and \( b \) – the slope of the line and its y-intercept – are unknowns to be determined.

- We define the error between the estimated line and the measured data to be the square of the distance between the line and the data at the \( x \) data points. Thus, our error is:

\[
E = \sum_{i=1}^{N} \left[ y_i - (mx_i + b) \right]^2
\]  \hspace{1cm} (4)

- If we minimize the error of equation (4) with respect to \( m \) and \( b \), we obtain the least-squares straight line fit to the data. We will not discuss the mathematical details of this step here – they are rather tedious.
• We can determine how well our straight line agrees with the data by calculating a *correlation coefficient*. The correlation coefficient, $r$, is calculated by:

$$r = \frac{1}{N-1} \sum_{i=1}^{N} \left( \frac{x_i - \bar{x}}{s_x} \right) \left( \frac{y_i - \bar{y}}{s_y} \right)$$

where $\bar{x}$ and $\bar{y}$ are the means of the $x$ and $y$ data, respectively, and $s_x$ and $s_y$ are the standard deviations of the $x$ and $y$ data. The correlation coefficient is a number between -1 and 1 ($-1 \leq r \leq 1$); it essentially tells us how well our data agrees with the straight line curve fit. If all the data lie exactly on a straight line with positive slope, the correlation coefficient will be identically one ($r = 1$). If the data have noise or follow a nonlinear relationship, the correlation coefficient will be reduced. Data which are entirely uncorrelated have a correlation coefficient of zero ($r = 0$). A correlation coefficient of -1 simply means that there is a perfect negative relation between $x$ and $y$ – the data will lie on a straight line with negative slope.

Figure 3 provides several examples of data with various degrees of correlation.

![Figure 3. Example data and representative correlation coefficients.](image-url)
Using MATLAB for least squares curve fitting:

MATLAB’s `polyfit` function performs least-squares curve fitting. `polyfit` will fit an arbitrary-order polynomial to a set of data. Syntax for the function is

\[ p = \text{polyfit}(x,y,n) \]

where `x` and `y` are vectors containing the data to be fit, `n` is the order of polynomial to be fit to the data (a straight line is a first order polynomial, so we will always set `n = 1`). The function returns a vector containing the coefficients of the polynomial which provides a least-squares fit to the data. For `n = 1` a two-element vector will be returned; the first element of the vector will be the slope of the line (\(m\), in equation (3)) and the second element will be the y-intercept of the line (\(b\), in equation (3)).

MATLAB’s `corrcoef` function provides the correlation coefficient of two data sets. Possible syntax for using this function is:

\[ r = \text{corrcoef}(x,y) \]

where `x` and `y` are vectors containing the data. This use of the function will return a 2x2 matrix; it will have the following form:

\[ r = \begin{bmatrix} r_{xx} & r_{xy} \\ r_{yx} & r_{yy} \end{bmatrix} \]

This matrix provides correlations between all possible combinations of the data provided to the function. \(r_{xx}\) is the correlation between the \(x\) data and itself. Likewise, \(r_{yy}\) is the correlation between the \(y\) data and itself. Since data is always perfectly correlated with itself, \(r_{xx} = r_{yy} = 1\) always. \(r_{xy}\) is the correlation between the \(x\) data and the \(y\) data, and \(r_{yx}\) is the correlation between the \(y\) data and the \(x\) data. For us, \(r_{xy} = r_{yx}\). Thus, either the \(r_{xy}\) or \(r_{yx}\) terms will give us the correlation coefficient as defined in equation (5).

Pre-lab: None

Lab Procedures:

1. Connect the circuit shown in Figure 1. Use a 10\(\Omega\) resistor and a variable power supply. Use an ohmmeter to measure the actual resistance of your resistor and record the value in your lab notebook.

2. Vary the supply voltage \(v_s\) from 0V to approximately 2V. Measure \(v_R\) and \(i_R\) for at least 10 different values of \(v_s\) over this range of applied voltage (e.g. measure \(v_R\) and \(i_R\) at approximately 0.2V increments in \(v_s\). Tabulate the measured values of \(v_R\) and \(i_R\) in your lab notebook.

3. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

Note: we will use a linear regression of these data in the post-lab exercises to estimate resistance
III. Nonlinear Resistance

General Discussion:

In this part of the lab assignment, we will apply excessive voltage levels to our resistor from part II. This will result in higher power dissipation than the resistor is rated for, which will in turn cause the resistor to operate as a nonlinear device. The resistor will fail as a result of this test – you may dispose of the resistor after this experiment.

Caution:

Components may become very hot during this experiment. It is our intention during this experiment to cause failure of a resistor. During this process, the resistor will become hot and may emit smoke. Do not touch the resistor during this experiment. Conduct the experiment in a well-ventilated area.

Pre-lab: None

Lab Procedures:

1. Use the same experimental circuit as shown in Figure 1 and used in part II of this experiment. Starting with a supply voltage of 2V ($v_s = 2V$), increase the supply voltage at approximately 0.5V increments. Record $v_R$ and $i_R$ at each value of $v_s$. Continue to increase the supply voltage and record the resistor voltage and current until $v_s$ is approximately 7V.

   Reminder: When measuring current, always make a preliminary measurement using the highest-current port; if no current registers when using this port, then switch to the lower-current port. It is possible that you will blow out a fuse in your DMM if you use the low-current port of your DMM over the entire range of voltages in this lab assignment, since we may provide fairly high currents to the resistor.

2. Calculate a resistance value for each of the voltage-current values recorded above. Does the resistance remain constant over the entire range of the data? If we define the percentage change in resistance as:

   \[
   \text{% change} = \left( \frac{R_{\text{nominal}} - R}{R_{\text{nominal}}} \right) \times 100
   \]

   where $R_{\text{nominal}}$ is the nominal (meaning, literally, “in name only”) value of resistance and $R$ is the actual resistance value – in this case, the resistance that differs most from the resistor’s “nominal” value, over the measured data range. As your nominal resistance, you can use either the 10Ω value specified in the lab instructions or the resistance measured in part II of this assignment. (Just be sure your lab notebook what your “nominal resistance” is.)

3. Turn off the power supply and allow the resistor to cool down. After the resistor has cooled, apply $v_s = 1V$ to the resistor and measure $v_R$ and $i_R$. Calculate a resistance value from this data. Verify this resistance value with your ohmmeter. Does the resistance value agree with the original resistance value of the resistor? What is the percent change in resistance? Demonstrate operation of your circuit to the Teaching Assistant Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.
IV. Dusk-to-Dawn Light

General Discussion:

In this part of the lab assignment, we will create a light-sensitive lighting system. A photocell – a light-sensitive resistor – will be used to sense the ambient light level. A Bipolar Junction Transistor (BJT) will be used as a switch to turn on a light-emitting diode when the ambient light level becomes low. The circuit we will use is shown in Figure 4.

![Dusk-to-dawn lighting circuit diagram](image)

Figure 4. Dusk-to-dawn lighting circuit.

There are several unfamiliar components in the circuit of Figure 4; a photocell, an LED, and a BJT. A detailed understanding of the operation of these components is beyond the scope of this course, but a brief overview of their operating characteristics is provided below. Later courses in a typical electrical engineering curriculum will provide background information necessary to fully understand these components.

**Diodes and LEDs:**

Diodes are two-terminal semiconductor devices that conduct current in only one direction. The terminals of a diode are called the anode and the cathode; diodes are intended to conduct current from the anode to the cathode. Diodes have a minimum threshold voltage (or \( V_{th} \), usually around 0.7V) that must be present between the anode and cathode in order for current to flow. If the anode voltage is not at least \( V_{th} \) greater than the cathode voltage, no current will flow. Likewise, if the cathode voltage is greater than the anode voltage, the diode is said to be reverse-biased and no current will flow. In an ideal diode, if the diode voltage equals the threshold voltage (plus a small amount), then unlimited current can flow without causing the voltage across the diode to increase. And, if the diode is reversed-biased, no current will flow regardless of reverse-voltage magnitude.

As with diodes, LED's are two-terminal semiconductor devices that conduct current in only one direction (from the anode to the cathode). The small LED chips are secured inside a plastic housing, and they emit light at a given frequency when a small electric current (typically 10mA to 25mA) flows through them. When the voltage difference across the LED exceeds the threshold voltage of the LED, current flows through the LED and light is emitted. If the LED voltage is less
than the threshold voltage, no current flows and no light is emitted. LEDs are available in a number of colors; the Digilent analog parts kit contains red, yellow, and green LEDs.

Since LEDs are polarized devices, they must be placed in the circuit with the correct orientation; the anode must be at higher voltage potential than the cathode in order for the diode to emit light. An LED schematic symbol is shown in Figure 5 below, together with a sketch of a physical LED. The anode and cathode on a physical LED can be identified because the anode pin is longer than the cathode pin and the cathode side of the plastic diffusion lens is typically slightly flattened.

![LED schematic symbol and physical appearance](image)

**Figure 5.** LED schematic symbol and physical appearance.

**Bipolar Junction Transistors (BJTs):**

In Lab 0, we used a MOSFET as a voltage controlled current source. Bipolar Junction Transistors, BJTs, are also conveniently modeled as dependent sources. Like MOSFETs, BJTs are three-terminal devices; the terminals of a BJT are called the base (B), the collector (C), and the emitter (E). The symbol commonly used to represent the type of BJT we will be using is shown in Figure 6(a). Our circuit employs a PN2222 BJT; the physical appearance of this BJT is shown in Figure 6(b), along with the relative locations of the base, collector, and emitter for that BJT.

![BJT symbol and physical appearance of PN2222 BJT](image)

**Figure 6.** BJT symbol and physical appearance of PN2222 BJT.

An extremely simplified discussion of a BJT’s operation is as follows: application of voltage to the base of the BJT allows current to flow from the collector to the emitter of the BJT. Typically the current flowing into the base of the BJT is much smaller than the collector and emitter currents.
Thus, the BJT can be conceptualized as a current controlled current source. Thus, if a power supply is connected to the collector of the BJT, the base voltage of the BJT can be used to control BJT’s emitter current: increasing the base voltage typically increases the emitter current. In the case of two relatively discrete values of base voltage, the BJT can act as a switch; low base voltages turn off the switch (the emitter current is zero) while high base voltages turn the switch on (the emitter current is non-zero).

Photocell:

Photocells (sometimes called photoresistors or photoconductors) are devices whose resistance changes according to the light intensity applied to the sensor. The photocells in the Digilent analog parts kits have resistances which vary from about 5KΩ at relatively high light levels to about 20KΩ at relatively low light levels. A variable resistor is commonly indicated on a circuit schematic by a resistor symbol with an arrow through it, as shown in Figure 7.

![Figure 7. Variable resistor circuit symbol.](image)

Pre-lab:

Apply KVL around the outer loop of the circuit of Figure 4 (as shown in Figure 8) to determine the voltage $V_B$ for photocell resistances of 5KΩ and 20KΩ. You may assume that the current into the base of the BJT is negligible.

![Figure 8. Circuit analysis to determine BJT base voltage.](image)
Lab Procedures:

1. Construct the circuit shown in Figure 4. The LED should not light under normal light levels. Using a DMM, measure the base voltage of the BJT (V_B in Figure 8) and the voltage difference across the diode (V_D in Figure 8). Record these voltages in your lab notebook and compare them with your calculated values from the prelab.

2. Cover the photocell; the LED should light up. Using a DMM, measure the base voltage of the BJT (V_B in Figure 8) and the voltage difference across the diode (V_D in Figure 8). Record these voltages in your lab notebook and compare them with your calculated values from the prelab.

3. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

V. Post-Lab Exercises

1. Determine a least-squares curve fit of the v_R vs. i_R data measured in Part II. Plot the resulting line and the measured v_R vs. i_R data on the same graph. Comment on your results. Calculate a correlation coefficient for the data. Comment on your correlation coefficient relative to the qualitative agreement between the line and the data as shown on your plot.

2. Plot of the v_R vs. i_R data acquired in Part III. Comment on how well your data are approximated by a single straight line, over the entire voltage range. Discuss how this compares with your understanding of linear phenomena.

3. Calculate the power dissipated by the resistor for each data point acquired in Part III. At approximately what power level does the resistor begin to show nonlinear characteristics?